ASSESSING THE OUT OF SAMPLE FORECAST PERFORMANCE OF FINANCIAL DATA

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Abstract: The study focuses on the assessment of out of sample forecast performance of financial data with special reference to Gold Price data using Autoregressive Moving average (ARMA) model. The data used for the study was obtained from KITCO via the official website www.kitco.com. The data was subjected to various stationarity tests (graphical, correlogram and unit root test), the data was stationary at the second difference. Thereafter, various ARMA model was fitted from which ARMA(1,1) was chosen. The model fitted has a very powerful forecasting ability as Theil-U index value (0.997333) obtained was moderate, bias proportion (0.000003) and variance proportion (0.005521) almost tends to zero, and covariance proportion (0.994475) is very high. All these combined together enhanced good forecast performance and the forecast values for the year 2015-2020 were estimated monthly.

key words: ARMA, forecast, financial data, stationarity, correlogrom, stationarity

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1   Introduction

In all human endeavor be it business, industry, government, finance or any other organization, policy makers need to anticipate the future behavior of many important factors before making decision, such decisions lie on forecasts, for such forecasts to be accurate; a powerful forecast system is needed to make such decision. Statistical tests of a model's forecast performance are commonly conducted by splitting a given data set into an in-sample period used for initial parameter estimation and model selection and an out-of-sample period, used to evaluate forecast performance.

In sample forecast analysis could be explained as a means of estimating a model making use of the available data up to time T, and then compare the model's fitted values to the actual realizations.   Suppose we have a set of data say n (Y_t, X_{t+h}) where $h \in (1, 2, \ldots)$, and our goal is to build a model say $\left\{ \widehat{f}\left(X_{t+h}\right) \right\}$ to predict $Y_t \mid X_{t+h}$. The data set of X_{t+h} will be used for forecast the values for the future.

And an out-sample forecast performance is an estimate of a model based on data up to time T and including today, construct a forecast of tomorrow's value $Y_{t+h} Y_{t+1} Y_{t+h}$, wait until tomorrow, record the forecast error $e_{t+1} = Y_{t+1} - f\left(X_{t+1-h}\right)$, re-estimate the model, make a new forecast of Y_{t+2}, and so forth. At the end of this exercise, one would have a sample of forecast errors $\left\{ e_{t+1} \right\}$ which would be truly out-of-sample and would give a very realistic picture of the model's performance. Since this procedure is very time-consuming, people often resort to "pseudo", or "simulated", out-of-sample analysis, which means to mimic the procedure described in the last paragraph, using some historical date $T_0 < T$, rather than today's date T, as a starting point. The resulting forecasting errors $\left\{ e_t \right\} T_t = T_{0+1}$ are then used to get an estimate of the model's out-of-sample forecasting ability.

In the past decades, so many researches bothering on either in - or out - of samples forecast performances have been conducted prominent among them are Sherman (1982, 1983), Ariovich (1983), Fortune (1987), Dooley et al. (1995), Sjaastad and Scacciallani (1996), Lucey et al. (2006), and Wang and Lee (2011).

On the other hand Tests of the market efficiency hypothesis for gold markets have been undertaken by Tschoegl (1980), Solt and Swanson (1981), Ho (1985), Basu and Clouse (1993), and Smith (2002). Jaffe (1989), Chua et al. (1990), Ciner (2001), Michaud et al. (2006), Hillier et al. (2006), McCown and Zimmerman (2006), Baur and Lucey (2010), Baur and McDermott (2010), and Ciner et al. (2010).

MATHEMATICAL PRELIMINARIES

The basic autoregressive model for a series X is,

$$X_t = C_t + \sum_{i=1}^{p} \phi X_{t-1} + \varepsilon_t$$

Where ε is a white noise error process and

$$\varphi X_{t-1} = \varphi_1 X_{t-1} + \varphi_1 X_{t-2} \ldots \varphi_n X_{t-n}$$

The above is called $p-th$ order autoregressive process, or $AR(p)$.

On the other hand the basic moving average models represent X as a function of current and lagged values of a white noise process.

$$X_t = \sum_{i=1}^{q} \theta_i X_{t-1} + \varepsilon_t$$

Where ε is a white noise error process and

$$\theta_i X_{t-1} = \theta_1 X_{t-1} + \theta_2 X_{t-2} \ldots \theta_n X_{t-q}$$

The above is called $q-th$ order Moving average process, or $MA(q)$.

The combination of these two types of model is called an autoregressive moving average model $(ARMA)n,q$, where n is the order of the autoregressive part and q is the order of the moving average term.

ARMA FORECAST

Forecast using $ARMA(1,1)$ process

$$y_t = c + \theta_1 y_{t-1} + \varepsilon_t + \varphi \varepsilon_{t-1}$$

h=1 period ahead

$$y_{t+1} = c + \theta_1 y_t + \varepsilon_{t+1} + \varphi \varepsilon_t$$

Mean forecast

$$E\left(y_{t+1}\right) = \widehat{y}_{t+1} = c + \theta_1 y_t + \varepsilon_{t+1} + \varphi \varepsilon_t + \varphi_1 \varepsilon_t - c - \varphi_1 y_{t-1} + \varphi_1 \varepsilon_t = \varepsilon_{t+1}$$

Steps in modeling ARMA MODEL

ARMA modeling proceeds by a series of well-defined steps
.
1. Identification of the models - this consist of specifying the appropriate structure (AR, MA or ARMA) and order of the model. Identification is sometimes done by looking at plots of the ACF and PACF and Sometimes it is done by an automated iterative procedure.
2. The second step is to estimate the coefficients of the model. Coefficients of AR models can be estimated by least-squares regression. Estimation of parameters of MA and ARMA models.
3.   The third step is to check the model. This step is also called diagnostic checking, or verification

Data analysis

Descriptive statistics of monthly gold price
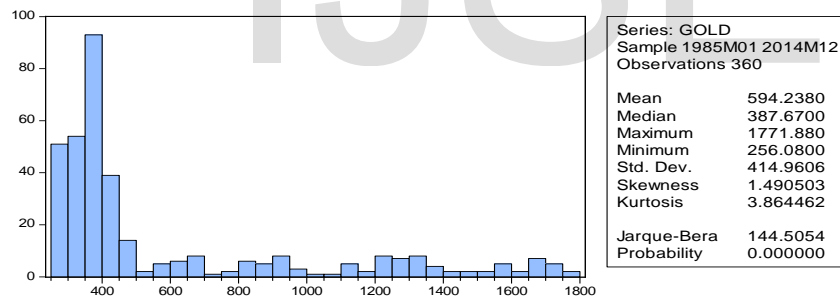
The above graph shows the descriptive statistics of the monthly gold price (USD) which can be seen not to be stationary with the chart of normal distribution skewed to the left with values of 1.490503, mean of 594.2380 and median 387.67.

For the assessment of stationarity of the data under study, the graphical look of the data is
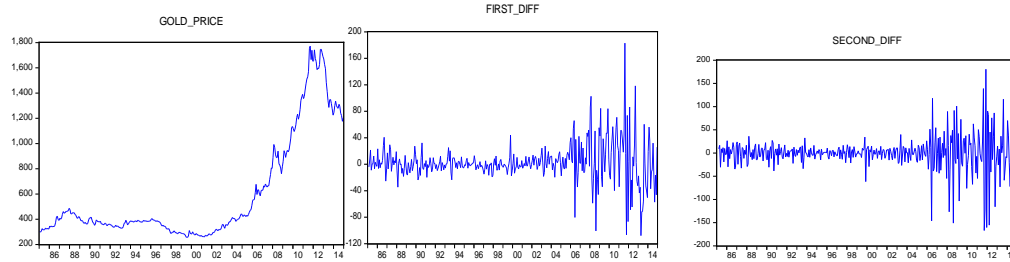
as follows:

Figure 1   Figure 2   Figure 3

Interpretation

The figures above are the time plot for the series. Figure 1 shows the time plot of the original series which shows that the data is not stationary and has a coefficient of determination R^2 of 0.155571 and the figure 2 shows the time plot for the first difference of the series which can say to be stationary but has a low coefficient of determination R^2 of 0.478488 and finally figure 3 shows the time plot for the second difference of the series which is also stationary and have a very good coefficient of determination R^2 of 0.805136 which confirmed that the series is stationary.

Unit root test (Augmented Dickey-Fuller)

The unit root tests for the series are given below

Null Hypothesis: GOLD_PRICE has a unit root
Exogenous: Constant
Lag Length: 11 (Automatic - based on AIC, maxlag=16)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.698271	0.8444
Test critical values:	1% level	-3.448943	
	5% level	-2.869629	
	10% level	-2.571148	

*MacKinnon (1996) one-sided p-values. $R^2 = 0.155571$

Table 1

Null Hypothesis: FIRST_DIFF has a unit root
Exogenous: Constant
Lag Length: 10 (Automatic - based on AIC, maxlag=16)

	t-Statistic	Prob.*

Augmented Dickey-Fuller test statistic		-4.214203	0.0007
Test critical values:	1% level	-3.448943	
	5% level	-2.869629	
	10% level	-2.571148	

*MacKinnon (1996) one-sided p-values. $R^2 = 0.478488$

Table 2

Null Hypothesis: SECOND_DIFF has a unit root
Exogenous: Constant
Lag Length: 14 (Automatic - based on AIC, maxlag=16)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.846067	0.0000
Test critical values:	1% level	-3.449220	
	5% level	-2.869750	
	10% level	-2.571213	

*MacKinnon (1996) one-sided p-values. $R^2 = 0.8051$

Table 3

Tables 1, 2 and 3 show the unit root test using AIC of the original, first difference and second difference of the series. Looking at the probability value of Augmented Dickey Fuller test of the original series (i.e. table 1), it is greater than 5% i.e. 0.8444>0.05 and the coefficient of determination is 0.1557 which implies that the series is not stationary. For table 2, the probability value is less than 5% i.e. 0.0007<0.05 but give a coefficient of determination of 0.478488 and finally table 3 is stationary because the probability value is less than 5% i.e. 0.0000< 0.05 and has a coefficient of determination of 0.805136

Correlogram

The correlogram of the series are shown below

Date: 09/02/15   Time: 14:16
Sample: 1985M01 2014M12
Included observations: 360

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|*******	.|*******	1	0.994	0.994	358.61	0.000
.|*******	.|.	2	0.987	-0.060	713.36	0.000
.|*******	.|.	3	0.980	-0.029	1064.0	0.000
.|*******	.|.	4	0.973	-0.038	1410.2	0.000
.|*******	.|.	5	0.965	-0.037	1751.7	0.000
.|*******	*|.	6	0.956	-0.075	2087.9	0.000
.|*******	.|.	7	0.947	-0.003	2418.6	0.000
.|*******	.|.	8	0.937	-0.019	2743.8	0.000

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|*******	.|. |	9	0.928	0.017	3063.7	0.000
.|*******	.|. |	10	0.919	0.004	3378.2	0.000
.|*******	.|. |	11	0.910	0.002	3687.4	0.000
.|******|	.|. |	12	0.901	-0.038	3991.1	0.000
.|******|	.|. |	13	0.891	-0.015	4289.2	0.000
.|******|	.|. |	14	0.881	-0.064	4581.3	0.000
.|******|	.|. |	15	0.870	-0.041	4867.2	0.000
.|******|	.|. |	16	0.858	-0.057	5146.3	0.000

Figure 4

Date: 09/02/15   Time: 14:18
Sample: 1985M01 2014M12
Included observations: 359

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|* |	.|* |	1	0.187	0.187	12.656	0.000
.|. |	.|. |	2	-0.025	-0.063	12.890	0.002
.|* |	.|* |	3	0.074	0.095	14.905	0.002
.|. |	*|. |	4	-0.040	-0.079	15.496	0.004
.|* |	.|* |	5	0.155	0.198	24.279	0.000
.|* |	.|. |	6	0.082	-0.008	26.735	0.000
.|. |	.|. |	7	0.038	0.063	27.266	0.000
*|. |	*|. |	8	-0.074	-0.140	29.263	0.000
*|. |	.|. |	9	-0.094	-0.025	32.550	0.000
.|. |	.|. |	10	0.059	0.041	33.845	0.000
.|* |	.|* |	11	0.187	0.193	46.902	0.000
.|* |	.|. |	12	0.092	-0.000	50.039	0.000
.|. |	.|. |	13	0.026	0.046	50.288	0.000
.|. |	.|. |	14	0.021	0.003	50.453	0.000
.|. |	.|* |	15	0.064	0.095	51.988	0.000
.|* |	.|. |	16	0.130	0.043	58.393	0.000

Figure 5

Date: 09/02/15   Time: 14:18
Sample: 1985M01 2014M12
Included observations: 358

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
***|. |	***|. |	1	-0.366	-0.366	48.408	0.000
*|. |	***|. |	2	-0.193	-0.378	61.953	0.000
.|* |	*|. |	3	0.134	-0.138	68.437	0.000
*|. |	***|. |	4	-0.193	-0.351	81.956	0.000
.|* |	*|. |	5	0.163	-0.103	91.639	0.000
.|. |	*|. |	6	-0.016	-0.158	91.731	0.000
.|. |	.|. |	7	0.042	0.047	92.366	0.000
.|. |	*|. |	8	-0.055	-0.077	93.461	0.000
*|. |	*|. |	9	-0.111	-0.133	97.987	0.000
.|. |	**|. |	10	0.014	-0.258	98.064	0.000
.|* |	.|. |	11	0.140	-0.047	105.34	0.000
.|. |	*|. |	12	-0.015	-0.084	105.43	0.000
.|. |	.|. |	13	-0.038	-0.037	105.96	0.000

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.|.    |              *|.    |        14   -0.030  -0.122   106.30   0.000
.|.    |               .|.    |        15   -0.016  -0.064   106.40   0.000
.|*    |               .|.    |        16    0.082  -0.012   108.95   0.000
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Figure 6

Figures 4 through 6 show the correlogram of the series. Figure 4 shows the correlogram of the original series with autocorrelation coefficient that starts with a value of 0.994 and decline very slowly to zero with a large value of lag k. Furthermore, figures 8 and 9 show no trend in the series hence, suggesting that the series is stationary.

Model fitting

As the descriptive statistics given in the beginning which reflects that the distribution of the gold price is a not normally distributed, suitable econometric modeling techniques are required for the gold price series of the study. to start with, we model the conditional mean process by autoregressive process AR(1) and moving average MA(1) and to do this, we draw a grid search table to denote the least Akaike information criterion (AIC) to discover the best fit as given in the table below:

MA(q) AR(p)	1	2	3	4	5	6	7
1	**9.529840**	9.673864	9.900214	9.879623	9.884243	9.895952	9.899618
2	9.543645	9.995917	9.996516	9.909693	9.956295	10.00463	10.00636
3	9.555281	9.977461	10.03014	9.995205	9.981497	10.03038	10.02536
4	9.546742	9.934911	10.00497	10.00286	10.00493	10.00993	10.00066
5	9.547699	9.951244	9.989599	10.00245	10.02669	10.02659	10.01995
6	9.562653	9.989522	10.03819	10.00384	10.02919	10.05701	10.05542
7	9.566230	9.978813	10.03656	10.00362	10.02408	10.05760	10.05076

Table 4

The table above shows both processes that demonstrate correlation residuals. Among all the different models applied to the data, ARMA(1,1) appears to be relatively better fit on the basis of Akaike Information Criterion. The results of ARMA(1,1) are shown below

Dependent Variable: SECOND_DIFF
Method: Least Squares
Date: 09/02/15   Time: 14:46
Sample (adjusted): 1985M04 2014M12
Included observations: 357 after adjustments
Convergence achieved after 14 iterations
MA Backcast: 1985M03

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.180705	0.052557	3.438267	0.0007
MA(1)	-0.990356	0.006011	-164.7706	0.0000

Table 5

The Correlogram for the above model i.e. ARMA(1,1) residuals is shown in figure 7 below

Actual and ARMA Model Correlogram
Specification: SECOND_DIFF AR(1) MA(1)
Date: 09/02/15   Time: 14:52
Sample: 1985M01
2014M12
Included observations: 357

	Autocorrelation				Partial Autocorrelation		
	Actual	Model	Difference		Actual	Model	Difference
0	1.000	1.000	0.000	0	1.000	1.000	0.000
1	-0.367	-0.410	0.043	1	-0.367	-0.410	0.043
2	-0.193	-0.074	-0.119	2	-0.378	-0.291	-0.088
3	0.133	-0.013	0.147	3	-0.138	-0.225	0.087
4	-0.193	-0.002	-0.190	4	-0.351	-0.184	-0.167
5	0.163	-0.000	0.163	5	-0.104	-0.155	0.051
6	-0.016	-0.000	-0.015	6	-0.157	-0.134	-0.023
7	0.041	-0.000	0.041	7	0.047	-0.118	0.165
8	-0.054	-0.000	-0.054	8	-0.076	-0.106	0.029
9	-0.111	-0.000	-0.111	9	-0.134	-0.096	-0.038
10	0.014	-0.000	0.014	10	-0.259	-0.087	-0.171
11	0.141	-0.000	0.141	11	-0.047	-0.080	0.033
12	-0.016	-0.000	-0.016	12	-0.084	-0.074	-0.010
13	-0.038	-0.000	-0.038	13	-0.037	-0.069	0.032
14	-0.030	-0.000	-0.030	14	-0.123	-0.065	-0.058
15	-0.016	-0.000	-0.016	15	-0.064	-0.061	-0.003
16	0.082	-0.000	0.082	16	-0.012	-0.057	0.045

Figure 7

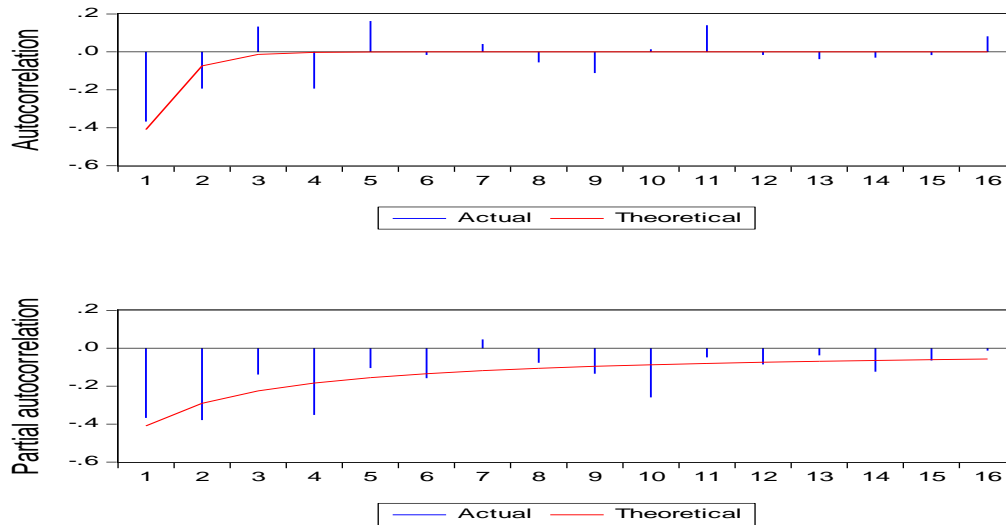The graph of the ARMA(1,1) residuals is given below

Figure 8

From the table above, the correlogram of both autocorrelation and partial autocorrelation give the impression that the estimated residuals are purely random.

Forecast analysis

Forecast performance of the fitted ARMA (1,1) model of gold price is investigated through the mean absolute error (MAE), root mean square error (RMSE), mean absolute percentage error (MAPE) and Theil inequality coefficient. The results are shown in the table 6 below: -

Root Mean Squared Error	36.54384
Mean Absolute Error	21.95672
Mean Absolute Percentage Error	100.0336
Theil Inequality Coefficient	0.997333
Bias Proportion	0.000003
Variance Proportion	0.005521
Covariance Proportion	0.994475

Table 6

The value of Theil-U inequality obtained is 0.997333 showing that the model fit is good. Looking at the bias proportion (0.000003) and variance proportion (0.005521). These two indices are very close to zero, the implication of this is that the series under study has a little or no bias error. The variance proportion is a bit close to zero implying a better fit. The covariance

proportion tends to one (0.994475) implying that this model will be very good for use for forecasting purpose.

Out of sample forecast

YEAR MONTH	2015	2016	2017	2018	2019	2020
JANUARY	1212.841	1250.128	1288.561	1328.175	1369.008	1411.096
FEBRUARY	1215.905	1253.286	1291.816	1331.531	1372.467	1414.661
MARCH	1218.977	1256.453	1295.08	1334.895	1375.934	1418.235
APRIL	1222.057	1259.627	1298.352	1338.268	1379.41	1421.818
MAY	1225.145	1262.81	1301.632	1341.649	1382.896	1425.41
JUNE	1228.24	1266	1304.921	1345.039	1386.39	1429.012
JULY	1231.343	1269.199	1308.218	1348.437	1389.892	1432.622
AUGUST	1234.454	1272.405	1311.523	1351.844	1393.404	1436.242
SEPTEMBER	1237.573	1275.62	1314.837	1355.259	1396.924	1439.87
OCTOBER	1240.7	1278.843	1318.159	1358.683	1400.454	1443.508
NOVEMBER	1243.835	1282.074	1321.489	1362.116	1403.992	1447.155
DECEMBER	1246.977	1285.313	1324.828	1365.558	1407.539	1450.812

Table 7

Findings and conclusion

We presented the data inform of a time plot, which is the plotting of price against time. The time plot shows that there is a relative upward and downward movement, this means that there is an increase and decrease in the number of gold price with time and this could result from inflation in the economy.

The time plot also shows that there is no seasonal variation, since the changes in the plot does not show annual change. The variation that exists is systematic and follows the cyclic variation and the knowledge of variation over some time would make the business, organisation, individuals to prepare and to adjust for the time boom, recession and recovery. Also, the gradual increase in the trend over years indicates that there would be increase in price in the nearest future.

We proceeded to the data analysis stage where ARMA (1, 1) model was chosen as it produced the least Akaike Information Criterion (AIC) value. Thereafter, we subjected the result obtained to forecast the future price of Gold.

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